

# Foundations for College Mathematics, Grade 12

College Preparation

MAP4C

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This course enables students to broaden their understanding of real-world applications of mathematics. Students will analyse data using statistical methods; solve problems involving applications of geometry and trigonometry; solve financial problems connected with annuities, budgets, and renting or owning accommodation; simplify expressions; and solve equations. Students will reason mathematically and communicate their thinking as they solve multi-step problems. This course prepares students for college programs in areas such as business, health sciences, and human services, and for certain skilled trades.

**Prerequisite:** Foundations for College Mathematics, Grade 11, College Preparation, or Functions and Applications, Grade 11, University/College Preparation

## MATHEMATICAL PROCESS EXPECTATIONS

The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

- Problem Solving**
  - develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
- Reasoning and Proving**
  - develop and apply reasoning skills (e.g., use of inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
  - demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
- Reflecting**
- Selecting Tools and Computational Strategies**
  - select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
- Connecting**
  - make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
  - create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;
- Representing**
- Communicating**
  - communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

# A. MATHEMATICAL MODELS

## OVERALL EXPECTATIONS

By the end of this course, students will:

1. evaluate powers with rational exponents, simplify algebraic expressions involving exponents, and solve problems involving exponential equations graphically and using common bases;
2. describe trends based on the interpretation of graphs, compare graphs using initial conditions and rates of change, and solve problems by modelling relationships graphically and algebraically;
3. make connections between formulas and linear, quadratic, and exponential relations, solve problems using formulas arising from real-world applications, and describe applications of mathematical modelling in various occupations.

## SPECIFIC EXPECTATIONS

### 1. Solving Exponential Equations

By the end of this course, students will:

- 1.1 determine, through investigation (e.g., by expanding terms and patterning), the exponent laws for multiplying and dividing algebraic expressions involving exponents [e.g.,  $(x^3)(x^2)$ ,  $x^3 \div x^5$ ] and the exponent law for simplifying algebraic expressions involving a power of a power [e.g.  $(x^6y^3)^2$ ]
- 1.2 simplify algebraic expressions containing integer exponents using the laws of exponents  
**Sample problem:** Simplify  $\frac{a^2b^5c^5}{ab^{-3}c^4}$  and evaluate for  $a = 8$ ,  $b = 2$ , and  $c = -30$ .
- 1.3 determine, through investigation using a variety of tools (e.g., calculator, paper and pencil, graphing technology) and strategies (e.g., patterning; finding values from a graph; interpreting the exponent laws), the value of a power with a rational exponent (i.e.,  $x^{\frac{m}{n}}$ , where  $x > 0$  and  $m$  and  $n$  are integers)  
**Sample problem:** The exponent laws suggest that  $4^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 4^1$ . What value would you assign to  $4^{\frac{1}{2}}$ ? What value would you assign to  $27^{\frac{1}{3}}$ ? Explain your reasoning. Extend your reasoning to make a generalization about the meaning of  $x^{\frac{1}{n}}$ , where  $x > 0$  and  $n$  is a natural number.
- 1.4 evaluate, with or without technology, numerical expressions involving rational exponents and rational bases [e.g.,  $2^{-3}$ ,  $(-6)^3$ ,  $4^{\frac{1}{2}}$ ,  $1.01^{120}$ ]\*
- 1.5 solve simple exponential equations numerically and graphically, with technology (e.g., use systematic trial with a scientific calculator to determine the solution to the equation  $1.05^x = 1.276$ ), and recognize that the solutions may not be exact  
**Sample problem:** Use the graph of  $y = 3^x$  to solve the equation  $3^x = 5$ .
- 1.6 solve problems involving exponential equations arising from real-world applications by using a graph or table of values generated with technology from a given equation [e.g.,  $h = 2(0.6)^n$ , where  $h$  represents the height of a bouncing ball and  $n$  represents the number of bounces]  
**Sample problem:** Dye is injected to test pancreas function. The mass,  $R$  grams, of dye remaining in a healthy pancreas after  $t$  minutes is given by the equation  $R = I(0.96)^t$ , where  $I$  grams is the mass of dye initially injected. If 0.50 g of dye is initially injected into a healthy pancreas, determine how much time elapses until 0.35 g remains by using a graph and/or table of values generated with technology.

\*The knowledge and skills described in this expectation are to be introduced as needed, and applied and consolidated, where appropriate, throughout the course.

- 1.7 solve exponential equations in one variable by determining a common base (e.g.,  $2^x = 32$ ,  $4^{5x-1} = 2^{2(x+11)}$ ,  $3^{5x+8} = 27^x$ )

**Sample problem:** Solve  $3^{5x+8} = 27^x$  by determining a common base, verify by substitution, and make connections to the intersection of  $y = 3^{5x+8}$  and  $y = 27^x$  using graphing technology.

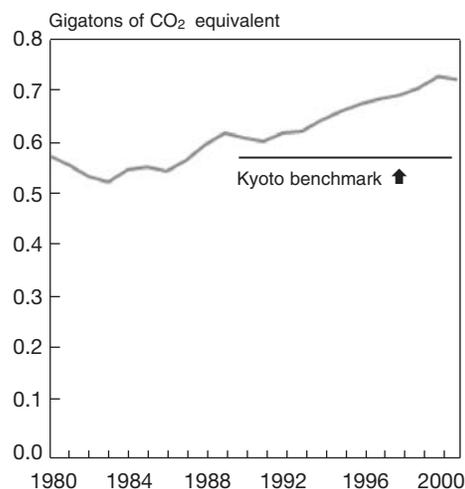
## 2. Modelling Graphically

By the end of this course, students will:

- 2.1 interpret graphs to describe a relationship (e.g., distance travelled depends on driving time, pollution increases with traffic volume, maximum profit occurs at a certain sales volume), using language and units appropriate to the context
- 2.2 describe trends based on given graphs, and use the trends to make predictions or justify decisions (e.g., given a graph of the men's 100-m world record versus the year, predict the world record in the year 2050 and state your assumptions; given a graph showing the rising trend in graduation rates among Aboriginal youth, make predictions about future rates)

**Sample problem:** Given the following graph, describe the trend in Canadian greenhouse gas emissions over the time period shown. Describe some factors that may have influenced these emissions over time. Predict the emissions today, explain your prediction using the graph and possible factors, and verify using current data.

Canadian Greenhouse Gas Emissions



Source: Environment Canada, Greenhouse Gas Inventory 1990-2001, 2003

- 2.3 recognize that graphs and tables of values communicate information about rate of change, and use a given graph or table of values for a relation to identify the units used to measure rate of change (e.g., for a distance–time graph, the units of rate of change are kilometres per hour; for a table showing earnings over time, the units of rate of change are dollars per hour)
- 2.4 identify when the rate of change is zero, constant, or changing, given a table of values or a graph of a relation, and compare two graphs by describing rate of change (e.g., compare distance–time graphs for a car that is moving at constant speed and a car that is accelerating)
- 2.5 compare, through investigation with technology, the graphs of pairs of relations (i.e., linear, quadratic, exponential) by describing the initial conditions and the behaviour of the rates of change (e.g., compare the graphs of amount versus time for equal initial deposits in simple interest and compound interest accounts)
- Sample problem:** In two colonies of bacteria, the population doubles every hour. The initial population of one colony is twice the initial population of the other. How do the graphs of population versus time compare for the two colonies? How would the graphs change if the population tripled every hour, instead of doubling?
- 2.6 recognize that a linear model corresponds to a constant increase or decrease over equal intervals and that an exponential model corresponds to a constant *percentage* increase or decrease over equal intervals, select a model (i.e., linear, quadratic, exponential) to represent the relationship between numerical data graphically and algebraically, using a variety of tools (e.g., graphing technology) and strategies (e.g., finite differences, regression), and solve related problems

**Sample problem:** Given the data table at the top of page 139, determine an algebraic model to represent the relationship between population and time, using technology. Use the algebraic model to predict the population in 2015, and describe any assumptions made.

| Years after 1955 | Population of Geese |
|------------------|---------------------|
| 0                | 5 000               |
| 10               | 12 000              |
| 20               | 26 000              |
| 30               | 62 000              |
| 40               | 142 000             |
| 50               | 260 000             |

### 3. Modelling Algebraically

By the end of this course, students will:

- 3.1 solve equations of the form  $x^n = a$  using rational exponents (e.g., solve  $x^3 = 7$  by raising both sides to the exponent  $\frac{1}{3}$ )
- 3.2 determine the value of a variable of degree no higher than three, using a formula drawn from an application, by first substituting known values and then solving for the variable, and by first isolating the variable and then substituting known values

*Sample problem:* Use the formula  $V = \frac{4}{3}\pi r^3$  to determine the radius of a sphere with a volume of  $1000 \text{ cm}^3$ .

- 3.3 make connections between formulas and linear, quadratic, and exponential functions [e.g., recognize that the compound interest formula,  $A = P(1 + i)^n$ , is an example of an exponential function  $A(n)$  when  $P$  and  $i$  are constant, and of a linear function  $A(P)$  when  $i$  and  $n$  are constant], using a variety of tools and strategies (e.g., comparing the graphs generated with technology when different variables in a formula are set as constants)

*Sample problem:* Which variable(s) in the formula  $V = \pi r^2 h$  would you need to set as a constant to generate a linear equation? A quadratic equation? Explain why you can expect the relationship between the volume and the height to be linear when the radius is constant.

- 3.4 solve multi-step problems requiring formulas arising from real-world applications (e.g., determining the cost of two coats of paint for a large cylindrical tank)
- 3.5 gather, interpret, and describe information about applications of mathematical modelling in occupations, and about college programs that explore these applications

## B. PERSONAL FINANCE

### OVERALL EXPECTATIONS

By the end of this course, students will:

1. demonstrate an understanding of annuities, including mortgages, and solve related problems using technology;
2. gather, interpret, and compare information about owning or renting accommodation, and solve problems involving the associated costs;
3. design, justify, and adjust budgets for individuals and families described in case studies, and describe applications of the mathematics of personal finance.

### SPECIFIC EXPECTATIONS

#### 1. Understanding Annuities

By the end of this course, students will:

- 1.1 gather and interpret information about annuities, describe the key features of an annuity, and identify real-world applications (e.g., RRSP, mortgage, RRIF, RESP)
- 1.2 determine, through investigation using technology (e.g., the TVM Solver on a graphing calculator; online tools), the effects of changing the conditions (i.e., the payments, the frequency of the payments, the interest rate, the compounding period) of an ordinary simple annuity (i.e., an annuity in which payments are made at the *end* of each period, and compounding and payment periods are the same) (e.g., long-term savings plans, loans)
 

*Sample problem:* Given an ordinary simple annuity with semi-annual deposits of \$1000, earning 6% interest per year compounded semi-annually, over a 20-year term, which of the following results in the greatest return: doubling the payments, doubling the interest rate, doubling the frequency of the payments and the compounding, or doubling the payment and compounding period?
- 1.3 solve problems, using technology (e.g., scientific calculator, spreadsheet, graphing calculator), that involve the amount, the present value, and the regular payment of an ordinary simple annuity

*Sample problem:* Using a spreadsheet, calculate the total interest paid over the life of a \$10 000 loan with monthly repayments over 2 years at 8% per year compounded monthly, and compare the total interest with the original principal of the loan.

- 1.4 demonstrate, through investigation using technology (e.g., a TVM Solver), the advantages of starting deposits earlier when investing in annuities used as long-term savings plans

*Sample problem:* If you want to have a million dollars at age 65, how much would you have to contribute monthly into an investment that pays 7% per annum, compounded monthly, beginning at age 20? At age 35? At age 50?

- 1.5 gather and interpret information about mortgages, describe features associated with mortgages (e.g., mortgages are annuities for which the present value is the amount borrowed to purchase a home; the interest on a mortgage is compounded semi-annually but often paid monthly), and compare different types of mortgages (e.g., open mortgage, closed mortgage, variable-rate mortgage)
- 1.6 read and interpret an amortization table for a mortgage

*Sample problem:* You purchase a \$200 000 condominium with a \$25 000 down payment, and you mortgage the balance at 6.5% per year compounded semi-annually over 25 years,

payable monthly. Use a given amortization table to compare the interest paid in the first year of the mortgage with the interest paid in the 25th year.

- 1.7 generate an amortization table for a mortgage, using a variety of tools and strategies (e.g., input data into an online mortgage calculator; determine the payments using the TVM Solver on a graphing calculator and generate the amortization table using a spreadsheet), calculate the total interest paid over the life of a mortgage, and compare the total interest with the original principal of the mortgage

- 1.8 determine, through investigation using technology (e.g., TVM Solver, online tools, financial software), the effects of varying payment periods, regular payments, and interest rates on the length of time needed to pay off a mortgage and on the total interest paid

**Sample problem:** Calculate the interest saved on a \$100 000 mortgage with monthly payments, at 6% per annum compounded semi-annually, when it is amortized over 20 years instead of 25 years.

## 2. Renting or Owning Accommodation

By the end of this course, students will:

- 2.1 gather and interpret information about the procedures and costs involved in owning and in renting accommodation (e.g., apartment, condominium, townhouse, detached home) in the local community
- 2.2 compare renting accommodation with owning accommodation by describing the advantages and disadvantages of each
- 2.3 solve problems, using technology (e.g., calculator, spreadsheet), that involve the fixed costs (e.g., mortgage, insurance, property tax) and variable costs (e.g., maintenance, utilities) of owning or renting accommodation

**Sample problem:** Calculate the total of the fixed and variable monthly costs that are associated with owning a detached house but that are usually included in the rent for rental accommodation.

## 3. Designing Budgets

By the end of this course, students will::

- 3.1 gather, interpret, and describe information about living costs, and estimate the living costs of different households (e.g., a family of four, including two young children; a single young person; a single parent with one child) in the local community
- 3.2 design and present a savings plan to facilitate the achievement of a long-term goal (e.g., attending college, purchasing a car, renting or purchasing a house)
- 3.3 design, explain, and justify a monthly budget suitable for an individual or family described in a given case study that provides the specifics of the situation (e.g., income; personal responsibilities; costs such as utilities, food, rent/mortgage, entertainment, transportation, charitable contributions; long-term savings goals), with technology (e.g., using spreadsheets, budgeting software, online tools) and without technology (e.g., using budget templates)
- 3.4 identify and describe the factors to be considered in determining the affordability of accommodation in the local community (e.g., income, long-term savings, number of dependants, non-discretionary expenses), and consider the affordability of accommodation under given circumstances
 

**Sample problem:** Determine, through investigation, if it is possible to change from renting to owning accommodation in your community in five years if you currently earn \$30 000 per year, pay \$900 per month in rent, and have savings of \$20 000.
- 3.5 make adjustments to a budget to accommodate changes in circumstances (e.g., loss of hours at work, change of job, change in personal responsibilities, move to new accommodation, achievement of a long-term goal, major purchase), with technology (e.g., spreadsheet template, budgeting software)
- 3.6 gather, interpret, and describe information about applications of the mathematics of personal finance in occupations (e.g., selling real estate, bookkeeping, managing a restaurant, financial planning, mortgage brokering), and about college programs that explore these applications

# C. GEOMETRY AND TRIGONOMETRY

## OVERALL EXPECTATIONS

By the end of this course, students will:

1. solve problems involving measurement and geometry and arising from real-world applications;
2. explain the significance of optimal dimensions in real-world applications, and determine optimal dimensions of two-dimensional shapes and three-dimensional figures;
3. solve problems using primary trigonometric ratios of acute and obtuse angles, the sine law, and the cosine law, including problems arising from real-world applications, and describe applications of trigonometry in various occupations.

## SPECIFIC EXPECTATIONS

### 1. Solving Problems Involving Measurement and Geometry

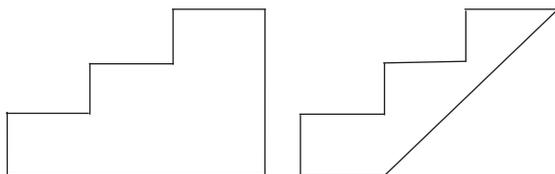
By the end of this course, students will:

- 1.1 perform required conversions between the imperial system and the metric system using a variety of tools (e.g., tables, calculators, online conversion tools), as necessary within applications
- 1.2 solve problems involving the areas of rectangles, triangles, and circles, and of related composite shapes, in situations arising from real-world applications

**Sample problem:** A car manufacturer wants to display three of its compact models in a triangular arrangement on a rotating circular platform. Calculate a reasonable area for this platform, and explain your assumptions and reasoning.

- 1.3 solve problems involving the volumes and surface areas of rectangular prisms, triangular prisms, and cylinders, and of related composite figures, in situations arising from real-world applications

**Sample problem:** Compare the volumes of concrete needed to build three steps that are 4 ft wide and that have the cross-sections shown below. Explain your assumptions and reasoning.

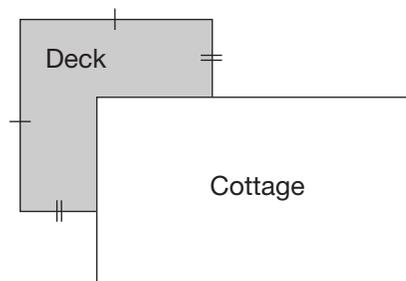


### 2. Investigating Optimal Dimensions

By the end of this course, students will:

- 2.1 recognize, through investigation using a variety of tools (e.g., calculators; dynamic geometry software; manipulatives such as tiles, geoboards, toothpicks) and strategies (e.g., modelling; making a table of values; graphing), and explain the significance of optimal perimeter, area, surface area, and volume in various applications (e.g., the minimum amount of packaging material, the relationship between surface area and heat loss)

**Sample problem:** You are building a deck attached to the second floor of a cottage, as shown below. Investigate how perimeter varies with different dimensions if you build the deck using exactly 48 1-m  $\times$  1-m decking sections, and how area varies if you use exactly 30 m of deck railing. *Note:* the entire outside edge of the deck will be railed.



**2.2** determine, through investigation using a variety of tools (e.g., calculators, dynamic geometry software, manipulatives) and strategies (e.g., modelling; making a table of values; graphing), the optimal dimensions of a two-dimensional shape in metric or imperial units for a given constraint (e.g., the dimensions that give the minimum perimeter for a given area)

**Sample problem:** You are constructing a rectangular deck against your house. You will use 32 ft of railing and will leave a 4-ft gap in the railing for access to stairs. Determine the dimensions that will maximize the area of the deck.

**2.3** determine, through investigation using a variety of tools and strategies (e.g., modelling with manipulatives; making a table of values; graphing), the optimal dimensions of a right rectangular prism, a right triangular prism, and a right cylinder in metric or imperial units for a given constraint (e.g., the dimensions that give the maximum volume for a given surface area)

**Sample problem:** Use a table of values and a graph to investigate the dimensions of a rectangular prism, a triangular prism, and a cylinder that each have a volume of  $64 \text{ cm}^3$  and the minimum surface area

### 3. Solving Problems Involving Trigonometry

By the end of this course, students will:

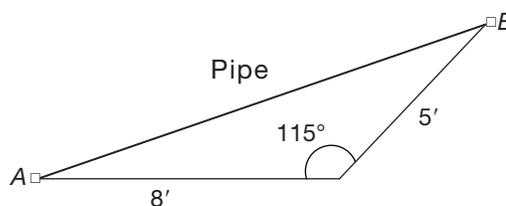
**3.1** solve problems in two dimensions using metric or imperial measurements, including problems that arise from real-world applications (e.g., surveying, navigation, building construction), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios, and of acute triangles using the sine law and the cosine law

**3.2** make connections between primary trigonometric ratios (i.e., sine, cosine, tangent) of obtuse angles and of acute angles, through investigation using a variety of tools and strategies (e.g., using dynamic geometry software to identify an obtuse angle with the same sine as a given acute angle; using a circular geoboard to compare congruent triangles; using a scientific calculator to compare trigonometric ratios for supplementary angles)

**3.3** determine the values of the sine, cosine, and tangent of obtuse angles

**3.4** solve problems involving oblique triangles, including those that arise from real-world applications, using the sine law (in non-ambiguous cases only) and the cosine law, and using metric or imperial units

**Sample problem:** A plumber must cut a piece of pipe to fit from  $A$  to  $B$ . Determine the length of the pipe.



**3.5** gather, interpret, and describe information about applications of trigonometry in occupations, and about college programs that explore these applications

**Sample problem:** Prepare a presentation to showcase an occupation that makes use of trigonometry, to describe the education and training needed for the occupation, and to highlight a particular use of trigonometry in the occupation.

## D. DATA MANAGEMENT

### OVERALL EXPECTATIONS

By the end of this course, students will:

1. collect, analyse, and summarize two-variable data using a variety of tools and strategies, and interpret and draw conclusions from the data;
2. demonstrate an understanding of the applications of data management used by the media and the advertising industry and in various occupations.

### SPECIFIC EXPECTATIONS

#### 1. Working With Two-Variable Data

By the end of this course, students will:

- 1.1** distinguish situations requiring one-variable and two-variable data analysis, describe the associated numerical summaries (e.g., tally charts, summary tables) and graphical summaries (e.g., bar graphs, scatter plots), and recognize questions that each type of analysis addresses (e.g., What is the frequency of a particular trait in a population? What is the mathematical relationship between two variables?)

*Sample problem:* Given a table showing shoe size and height for several people, pose a question that would require one-variable analysis and a question that would require two-variable analysis of the data.

- 1.2** describe characteristics of an effective survey (e.g., by giving consideration to ethics, privacy, the need for honest responses, and possible sources of bias, including cultural bias), and design questionnaires (e.g., for determining if there is a relationship between age and hours per week of Internet use, between marks and hours of study, or between income and years of education) or experiments (e.g., growth of plants under different conditions) for gathering two-variable data
- 1.3** collect two-variable data from primary sources, through experimentation involving observation or measurement, or from secondary sources (e.g., Internet databases, newspapers, magazines), and organize and store the data

using a variety of tools (e.g., spreadsheets, dynamic statistical software)

*Sample problem:* Download census data from Statistics Canada on age and average income, store the data using dynamic statistics software, and organize the data in a summary table.

- 1.4** create a graphical summary of two-variable data using a scatter plot (e.g., by identifying and justifying the dependent and independent variables; by drawing the line of best fit, when appropriate), with and without technology
- 1.5** determine an algebraic summary of the relationship between two variables that appear to be linearly related (i.e., the equation of the line of best fit of the scatter plot), using a variety of tools (e.g., graphing calculators, graphing software) and strategies (e.g., using systematic trials to determine the slope and  $y$ -intercept of the line of best fit; using the regression capabilities of a graphing calculator), and solve related problems (e.g., use the equation of the line of best fit to interpolate or extrapolate from the given data set)
- 1.6** describe possible interpretations of the line of best fit of a scatter plot (e.g., the variables are linearly related) and reasons for misinterpretations (e.g., using too small a sample; failing to consider the effect of outliers; interpolating from a weak correlation; extrapolating non-linearly related data)
- 1.7** determine whether a linear model (i.e., a line of best fit) is appropriate given a set of two-variable data, by assessing the correlation

between the two variables (i.e., by describing the type of correlation as positive, negative, or none; by describing the strength as strong or weak; by examining the context to determine whether a linear relationship is reasonable)

- 1.8** make conclusions from the analysis of two-variable data (e.g., by using a correlation to suggest a possible cause-and-effect relationship), and judge the reasonableness of the conclusions (e.g., by assessing the strength of the correlation; by considering if there are enough data)

## 2. Applying Data Management

By the end of this course, students will:

- 2.1** recognize and interpret common statistical terms (e.g., percentile, quartile) and expressions (e.g., accurate 19 times out of 20) used in the media (e.g., television, Internet, radio, newspapers)

- 2.2** describe examples of indices used by the media (e.g., consumer price index, S&P/TSX composite index, new housing price index) and solve problems by interpreting and using indices (e.g., by using the consumer price index to calculate the annual inflation rate)

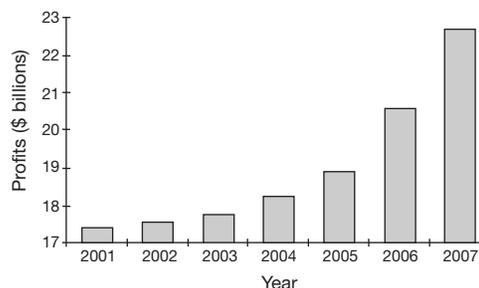
**Sample problem:** Use the new housing price index on E-STAT to track the cost of purchasing a new home over the past 10 years in the Toronto area, and compare with the cost in Calgary, Charlottetown, and Vancouver over the same period. Predict how much a new home that today costs \$200 000 in each of these cities will cost in 5 years.

- 2.3** interpret statistics presented in the media (e.g., the UN's finding that 2% of the world's population has more than half the world's wealth, whereas half the world's population has only 1% of the world's wealth), and

explain how the media, the advertising industry, and others (e.g., marketers, pollsters) use and misuse statistics (e.g., as represented in graphs) to promote a certain point of view (e.g., by making a general statement based on a weak correlation or an assumed cause-and-effect relationship; by starting the vertical scale on a graph at a value other than zero; by making statements using general population statistics without reference to data specific to minority groups)

- 2.4** assess the validity of conclusions presented in the media by examining sources of data, including Internet sources (i.e., to determine whether they are authoritative, reliable, unbiased, and current), methods of data collection, and possible sources of bias (e.g., sampling bias, non-response bias, a bias in a survey question), and by questioning the analysis of the data (e.g., whether there is any indication of the sample size in the analysis) and conclusions drawn from the data (e.g., whether any assumptions are made about cause and effect)

**Sample problem:** The headline that accompanies the following graph says "Big Increase in Profits". Suggest reasons why this headline may or may not be true.



- 2.5** gather, interpret, and describe information about applications of data management in occupations, and about college programs that explore these applications