Foundations for College Mathematics, Grade 11

College Preparation **MBF3C**

This course enables students to broaden their understanding of mathematics as a problem-solving tool in the real world. Students will extend their understanding of quadratic relations; investigate situations involving exponential growth; solve problems involving compound interest; solve financial problems connected with vehicle ownership; develop their ability to reason by collecting, analysing, and evaluating data involving one variable; connect probability and statistics; and solve problems in geometry and trigonometry. Students will consolidate their mathematical skills as they solve problems and communicate their thinking.

Prerequisite: Foundations of Mathematics, Grade 10, Applied

MATHEMATICAL PROCESS EXPECTATIONS

The mathematical processes are to be integrated into student learning in all areas of this course.

Throughout this course, students will:

Problem Solving	• develop, select, apply, compare, and adapt a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;
Reasoning and Proving	• develop and apply reasoning skills (e.g., use of inductive reasoning, deductive reasoning, and counter-examples; construction of proofs) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;
Reflecting	• demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);
Selecting Tools and Computational Strategies	• select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;
Connecting	• make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);
Representing	• create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;
Communicating	• communicate mathematical thinking orally, visually, and in writing, using precise mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

A. MATHEMATICAL MODELS

OVERALL EXPECTATIONS

By the end of this course, students will:

1. make connections between the numeric, graphical, and algebraic representations of quadratic relations, and use the connections to solve problems;
2. demonstrate an understanding of exponents, and make connections between the numeric, graphical, and algebraic representations of exponential relations;
3. describe and represent exponential relations, and solve problems involving exponential relations arising from real-world applications.

SPECIFIC EXPECTATIONS

1. Connecting Graphs and Equations of Quadratic Relations

By the end of this course, students will:

1.1 construct tables of values and graph quadratic relations arising from real-world applications (e.g., dropping a ball from a given height; varying the edge length of a cube and observing the effect on the surface area of the cube)

1.2 determine and interpret meaningful values of the variables, given a graph of a quadratic relation arising from a real-world application

Sample problem: Under certain conditions, there is a quadratic relation between the profit of a manufacturing company and the number of items it produces. Explain how you could interpret a graph of the relation to determine the numbers of items produced for which the company makes a profit and to determine the maximum profit the company can make.

1.3 determine, through investigation using technology, the roles of a, h, and k in quadratic relations of the form $y = a(x - h)^2 + k$, and describe these roles in terms of transformations on the graph of $y = x^2$ (i.e., translations; reflections in the x-axis; vertical stretches and compressions to and from the x-axis)

Sample problem: Investigate the graph $y = 3(x - h)^2 + 5$ for various values of h, using technology, and describe the effects of changing h in terms of a transformation.

1.4 sketch graphs of quadratic relations represented by the equation $y = a(x - h)^2 + k$ (e.g., using the vertex and at least one point on each side of the vertex; applying one or more transformations to the graph of $y = x^2$)

1.5 expand and simplify quadratic expressions in one variable involving multiplying binomials [e.g., $\left(\frac{1}{2}x + 1\right)(3x - 2)$] or squaring a binomial [e.g., $5(3x - 1)^2$], using a variety of tools (e.g., paper and pencil, algebra tiles, computer algebra systems)

1.6 express the equation of a quadratic relation in the standard form $y = ax^2 + bx + c$, given the vertex form $y = a(x - h)^2 + k$, and verify, using graphing technology, that these forms are equivalent representations

Sample problem: Given the vertex form $y = 3(x - 1)^2 + 4$, express the equation in standard form. Use technology to compare the graphs of these two forms of the equation.

1.7 factor trinomials of the form $ax^2 + bx + c$, where $a = 1$ or where a is the common factor, by various methods

1.8 determine, through investigation, and describe the connection between the factors of a quadratic expression and the x-intercepts of the graph of the corresponding quadratic relation

Sample problem: Investigate the relationship between the factored form of $3x^2 + 15x + 12$ and the x-intercepts of $y = 3x^2 + 15x + 12$.

1.9 solve problems, using an appropriate strategy (i.e., factoring, graphing), given equations of quadratic relations, including those that arise from real-world applications (e.g., break-even point)

Sample problem: On planet X, the height, h metres, of an object fired upward from the ground at 48 m/s is described by the equation $h = 48t - 16t^2$, where t seconds is the time since the object was fired upward. Determine the maximum height of the object, the times at which the object is 32 m above the ground, and the time at which the object hits the ground.

2. Connecting Graphs and Equations of Exponential Relations

By the end of this course, students will:

2.1 determine, through investigation using a variety of tools and strategies (e.g., graphing with technology; looking for patterns in tables of values), and describe the meaning of negative exponents and of zero as an exponent

2.2 evaluate, with and without technology, numeric expressions containing integer exponents and rational bases (e.g., 2^{-3}, 6^3, 3456^0, 1.03^{10})

2.3 determine, through investigation (e.g., by patterning with and without a calculator), the exponent rules for multiplying and dividing numerical expressions involving exponents [e.g., $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^2$], and the exponent rule for simplifying numerical expressions involving a power of a power [e.g., $\left(5^3\right)^2$]

2.4 graph simple exponential relations, using paper and pencil, given their equations [e.g., $y = 2^x$, $y = 10^x$, $y = \left(\frac{1}{2}\right)^x$]

2.5 make and describe connections between representations of an exponential relation (i.e., numeric in a table of values; graphical; algebraic)

2.6 distinguish exponential relations from linear and quadratic relations by making comparisons in a variety of ways (e.g., comparing rates of change using finite differences in tables of values; inspecting graphs; comparing equations), within the same context when possible (e.g., simple interest and compound interest, population growth)

Sample problem: Explain in a variety of ways how you can distinguish exponential growth represented by $y = 2^x$ from quadratic growth represented by $y = x^2$ and linear growth represented by $y = 2x$.

3. Solving Problems Involving Exponential Relations

By the end of this course, students will:

3.1 collect data that can be modelled as an exponential relation, through investigation with and without technology, from primary sources, using a variety of tools (e.g., concrete materials such as number cubes, coins; measurement tools such as electronic probes), or from secondary sources (e.g., websites such as Statistics Canada, E-STAT), and graph the data

Sample problem: Collect data and graph the cooling curve representing the relationship between temperature and time for hot water cooling in a porcelain mug. Predict the shape of the cooling curve when hot water cools in an insulated mug. Test your prediction.

3.2 describe some characteristics of exponential relations arising from real-world applications (e.g., bacterial growth, drug absorption) by using tables of values (e.g., to show a constant ratio, or multiplicative growth or decay) and graphs (e.g., to show, with technology, that there is no maximum or minimum value)

3.3 pose problems involving exponential relations arising from a variety of real-world applications (e.g., population growth, radioactive decay, compound interest), and solve these and other such problems by using a given graph or a graph generated with technology from a given table of values or a given equation

Sample problem: Given a graph of the population of a bacterial colony versus time, determine the change in population in the first hour.

3.4 solve problems using given equations of exponential relations arising from a variety of real-world applications (e.g., radioactive decay, population growth, height of a bouncing ball, compound interest) by substituting values for the exponent into the equations

Sample problem: The height, h metres, of a ball after n bounces is given by the equation $h = 2(0.6)^n$. Determine the height of the ball after 3 bounces.

B. PERSONAL FINANCE

OVERALL EXPECTATIONS

By the end of this course, students will:

1. compare simple and compound interest, relate compound interest to exponential growth, and solve problems involving compound interest;
2. compare services available from financial institutions, and solve problems involving the cost of making purchases on credit;
3. interpret information about owning and operating a vehicle, and solve problems involving the associated costs.

SPECIFIC EXPECTATIONS

1. Solving Problems Involving Compound Interest

By the end of this course, students will:

1.1 determine, through investigation using technology, the compound interest for a given investment, using repeated calculations of simple interest, and compare, using a table of values and graphs, the simple and compound interest earned for a given principal (i.e., investment) and a fixed interest rate over time

Sample problem: Compare, using tables of values and graphs, the amounts after each of the first five years for a $1000 investment at 5% simple interest per annum and a $1000 investment at 5% interest per annum, compounded annually.

1.2 determine, through investigation (e.g., using spreadsheets and graphs), and describe the relationship between compound interest and exponential growth

1.3 solve problems, using a scientific calculator, that involve the calculation of the amount, A (also referred to as future value, FV), and the principal, P (also referred to as present value, PV), using the compound interest formula in the form $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$]

Sample problem: Calculate the amount if $1000 is invested for 3 years at 6% per annum, compounded quarterly.

1.4 calculate the total interest earned on an investment or paid on a loan by determining the difference between the amount and the principal [e.g., using $I = A - P$ (or $I = FV - PV$)]

1.5 solve problems, using a TVM Solver on a graphing calculator or on a website, that involve the calculation of the interest rate per compounding period, i, or the number of compounding periods, n, in the compound interest formula $A = P(1 + i)^n$ [or $FV = PV(1 + i)^n$]

Sample problem: Use the TVM Solver on a graphing calculator to determine the time it takes to double an investment in an account that pays interest of 4% per annum, compounded semi-annually.

1.6 determine, through investigation using technology (e.g., a TVM Solver on a graphing calculator or on a website), the effect on the future value of a compound interest investment or loan of changing the total length of time, the interest rate, or the compounding period

Sample problem: Investigate whether doubling the interest rate will halve the time it takes for an investment to double.

2. Comparing Financial Services

By the end of this course, students will:

2.1 gather, interpret, and compare information about the various savings alternatives commonly available from financial institutions (e.g., savings and chequing accounts, term investments), the related costs (e.g., cost of cheques, monthly statement fees, early withdrawal penalties), and possible ways of reducing the costs (e.g., maintaining a minimum balance in a savings account; paying a monthly flat fee for a package of services)

2.2 gather and interpret information about investment alternatives (e.g., stocks, mutual funds, real estate, GICs, savings accounts), and compare the alternatives by considering the risk and the rate of return

2.3 gather, interpret, and compare information about the costs (e.g., user fees, annual fees, service charges, interest charges on overdue balances) and incentives (e.g., loyalty rewards; philanthropic incentives, such as support for Olympic athletes or a Red Cross disaster relief fund) associated with various credit cards and debit cards

2.4 gather, interpret, and compare information about current credit card interest rates and regulations, and determine, through investigation using technology, the effects of delayed payments on a credit card balance

2.5 solve problems involving applications of the compound interest formula to determine the cost of making a purchase on credit

Sample problem: Using information gathered about the interest rates and regulations for two different credit cards, compare the costs of purchasing a $1500 computer with each card if the full amount is paid 55 days later.

3. Owning and Operating a Vehicle

By the end of this course, students will:

3.1 gather and interpret information about the procedures and costs involved in insuring a vehicle (e.g., car, motorcycle, snowmobile) and the factors affecting insurance rates (e.g., gender, age, driving record, model of vehicle, use of vehicle), and compare the insurance costs for different categories of drivers and for different vehicles

Sample problem: Use automobile insurance websites to investigate the degree to which the type of car and the age and gender of the driver affect insurance rates.

3.2 gather, interpret, and compare information about the procedures and costs (e.g., monthly payments, insurance, depreciation, maintenance, miscellaneous expenses) involved in buying or leasing a new vehicle or buying a used vehicle

Sample problem: Compare the costs of buying a new car, leasing the same car, and buying an older model of the same car.

3.3 solve problems, using technology (e.g., calculator, spreadsheet), that involve the fixed costs (e.g., licence fee, insurance) and variable costs (e.g., maintenance, fuel) of owning and operating a vehicle

Sample problem: The rate at which a car consumes gasoline depends on the speed of the car. Use a given graph of gasoline consumption, in litres per 100 km, versus speed, in kilometres per hour, to determine how much gasoline is used to drive 500 km at speeds of 80 km/h, 100 km/h, and 120 km/h. Use the current price of gasoline to calculate the cost of driving 500 km at each of these speeds.

C. GEOMETRY AND TRIGONOMETRY

OVERALL EXPECTATIONS

By the end of this course, students will:

1. represent, in a variety of ways, two-dimensional shapes and three-dimensional figures arising from real-world applications, and solve design problems;

2. solve problems involving trigonometry in acute triangles using the sine law and the cosine law, including problems arising from real-world applications.

SPECIFIC EXPECTATIONS

1. Representing Two-Dimensional Shapes and Three-Dimensional Figures

By the end of this course, students will:

1.1 recognize and describe real-world applications of geometric shapes and figures, through investigation (e.g., by importing digital photos into dynamic geometry software), in a variety of contexts (e.g., product design, architecture, fashion), and explain these applications (e.g., one reason that sewer covers are round is to prevent them from falling into the sewer during removal and replacement)

Sample problem: Explain why rectangular prisms are often used for packaging.

1.2 represent three-dimensional objects, using concrete materials and design or drawing software, in a variety of ways (e.g., orthographic projections [i.e., front, side, and top views], perspective isometric drawings, scale models)

1.3 create nets, plans, and patterns from physical models arising from a variety of real-world applications (e.g., fashion design, interior decorating, building construction), by applying the metric and imperial systems and using design or drawing software

1.4 solve design problems that satisfy given constraints (e.g., design a rectangular berm that would contain all the oil that could leak from a cylindrical storage tank of a given height and radius), using physical models (e.g., built from popsicle sticks, cardboard, duct tape) or drawings (e.g., made using design or drawing software), and state any assumptions made

Sample problem: Design and construct a model boat that can carry the most pennies, using one sheet of 8.5 in. × 11 in. card stock, no more than five popsicle sticks, and some adhesive tape or glue.

2. Applying the Sine Law and the Cosine Law in Acute Triangles

By the end of this course, students will:

2.1 solve problems, including those that arise from real-world applications (e.g., surveying, navigation), by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios

2.2 verify, through investigation using technology (e.g., dynamic geometry software, spreadsheet), the sine law and the cosine law (e.g., compare, using dynamic geometry software, the ratios $\frac{a}{\sin A}$, $\frac{b}{\sin B}$, and $\frac{c}{\sin C}$ in triangle *ABC* while dragging one of the vertices);

2.3 describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles

2.4 solve problems that arise from real-world applications involving metric and imperial measurements and that require the use of the sine law or the cosine law in acute triangles

D. DATA MANAGEMENT

OVERALL EXPECTATIONS

By the end of this course, students will:

1. solve problems involving one-variable data by collecting, organizing, analysing, and evaluating data;
2. determine and represent probability, and identify and interpret its applications.

SPECIFIC EXPECTATIONS

1. Working With One-Variable Data

By the end of this course, students will:

1.1 identify situations involving one-variable data (i.e., data about the frequency of a given occurrence), and design questionnaires (e.g., for a store to determine which CDs to stock, for a radio station to choose which music to play) or experiments (e.g., counting, taking measurements) for gathering one-variable data, giving consideration to ethics, privacy, the need for honest responses, and possible sources of bias

Sample problem: One lane of a three-lane highway is being restricted to vehicles with at least two passengers to reduce traffic congestion. Design an experiment to collect one-variable data to decide whether traffic congestion is actually reduced.

1.2 collect one-variable data from secondary sources (e.g., Internet databases), and organize and store the data using a variety of tools (e.g., spreadsheets, dynamic statistical software)

1.3 explain the distinction between the terms *population* and *sample*, describe the characteristics of a good sample, and explain why sampling is necessary (e.g., time, cost, or physical constraints)

Sample problem: Explain the terms *sample* and *population* by giving examples within your school and your community.

1.4 describe and compare sampling techniques (e.g., random, stratified, clustered, convenience, voluntary); collect one-variable data from primary sources, using appropriate sampling techniques in a variety of real-world situations; and organize and store the data

1.5 identify different types of one-variable data (i.e., categorical, discrete, continuous), and represent the data, with and without technology, in appropriate graphical forms (e.g., histograms, bar graphs, circle graphs, pictographs)

1.6 identify and describe properties associated with common distributions of data (e.g., normal, bimodal, skewed)

1.7 calculate, using formulas and/or technology (e.g., dynamic statistical software, spreadsheet, graphing calculator), and interpret measures of central tendency (i.e., mean, median, mode) and measures of spread (i.e., range, standard deviation)

1.8 explain the appropriate use of measures of central tendency (i.e., mean, median, mode) and measures of spread (i.e., range, standard deviation)

Sample problem: Explain whether the mean or the median of your course marks would be the more appropriate representation of your achievement. Describe the additional information that the standard deviation of your course marks would provide.

1.9 compare two or more sets of one-variable data, using measures of central tendency and measures of spread

Sample problem: Use measures of central tendency and measures of spread to compare data that show the lifetime of an economy light bulb with data that show the lifetime of a long-life light bulb.

1.10 solve problems by interpreting and analysing one-variable data collected from secondary sources

2. Applying Probability

By the end of this course, students will:

2.1 identify examples of the use of probability in the media and various ways in which probability is represented (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1)

2.2 determine the theoretical probability of an event (i.e., the ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely), and represent the probability in a variety of ways (e.g., as a fraction, as a percent, as a decimal in the range 0 to 1)

2.3 perform a probability experiment (e.g., tossing a coin several times), represent the results using a frequency distribution, and use the distribution to determine the experimental probability of an event

2.4 compare, through investigation, the theoretical probability of an event with the experimental probability, and explain why they might differ

Sample problem: If you toss 10 coins repeatedly, explain why 5 heads are unlikely to result from every toss.

2.5 determine, through investigation using class-generated data and technology-based simulation models (e.g., using a random-number generator on a spreadsheet or on a graphing calculator), the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases (e.g., "If I simulate tossing a coin 1000 times using technology, the experimental probability that I calculate for tossing tails is likely to be closer to the theoretical probability than if I simulate tossing the coin only 10 times")

Sample problem: Calculate the theoretical probability of rolling a 2 on a number cube. Simulate rolling a number cube, and use the simulation to calculate the experimental probability of rolling a 2 over 10, 20, 30, …, 200 trials. Graph the experimental probability versus the number of trials, and describe any trend.

2.6 interpret information involving the use of probability and statistics in the media, and make connections between probability and statistics (e.g., statistics can be used to generate probabilities)